Determination of vector meson properties by matching resonance saturation to a constituent quark model

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Abstract. The properties of mesonic resonances can be calculated in terms of the low-energy coefficients of chiral perturbation theory (χPT) by extending unitarized χPT to higher energies. On the other hand, these low-energy coefficients can be calculated in two different models, namely i) by assuming resonance saturation and ii) within a constituent quark model. By matching the expressions of the two models combined with the results of unitarized χPT and the Weinberg sum rules, the properties of vector and axial-vector mesons can be calculated in the combined large-N_c and chiral limit.

PACS. 12.39.Fe Phenomenological quark models: Chiral Lagrangians – 14.40.Cs Properties of specific particles: Other mesons with $S = C = 0$, mass < 2.5 GeV

1 Introduction

What determines the properties of hadrons made from light quarks, chiral symmetry breaking (χSB) and/or confinement? It is nowadays common wisdom that the mechanism of χSB causes large constituent quark masses of the order of 300–400 MeV. Hence even without confinement the creation of a quark-antiquark pair is rather expensive. Therefore, the role of confinement for the description of light hadrons is at least diminished by the appearance of χSB [1]. This suggests that the properties of light hadrons are *quantitatively* determined by the effect of χSB. In such a scenario confinement enters only *qualitatively* by excluding non-white states and quark-antiquark thresholds. It is well known that such a picture works very well for pions (*e.g.*, [2] and references therein). It is the purpose of the present work to apply that picture to ρ- and a_1-mesons. One reason why one does not need confinement to describe the properties of pions can be found in the fact that the mass of these quasi-Goldstone bosons is much below the (constituent!) quark-antiquark threshold. This is of course different for other types of mesons. At first glance it seems that this messes up the line of reasoning given above. The point however is that *e.g.* ρ-mesons leave a trace also in the low-energy region much below their pole mass by mediating, *e.g.*, pion-pion interactions [3]. Hence the key idea is that on the one hand (χSB aspect) one can describe the low-energy region reliably by a (chiral!) quark model (without confinement) —as this region is far away from the quark-antiquark production threshold. On the
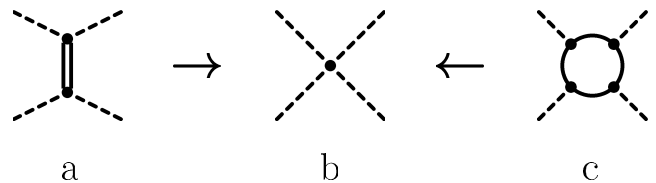

Fig. 1. Schematic view of the resonance saturation model (a) and the constituent quark model (c) and their respective low-energy reduction to χPT (b). The dashed lines denote Goldstone bosons, the double line mesonic resonances and the full lines quarks.

other hand (confinement aspect) the mesonic resonances are supposed to mediate the interactions in this low-energy regime. By matching corresponding expressions it should be possible to determine masses and coupling constants of mesonic resonances in terms of quark model expressions. This procedure is depicted schematically in fig. 1. For simplicity I work in the following in the combined large-N_c and chiral limit.[1]

2 Chiral perturbation theory and unitarization

At low energies QCD reduces to an effective theory where only the lightest mesons —the pseudoscalar Goldstone bosons— appear which interact with each other and with external sources. χSB demands that the meson interaction

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[1] To be specific I take the large-N_c limit first, *i.e.* neglect the chiral log's which are suppressed by $1/N_c$.

vanishes with vanishing energy. Therefore a systematic expansion in terms of the derivatives of the meson fields is possible. These considerations lead to the effective Lagrangian of chiral perturbation theory (χPT) [4]:

$$\mathcal{L}_{\chi\mathrm{PT}} = \mathcal{L}_1 + \mathcal{L}_2 + \text{higher order derivatives} \qquad (1)$$

with

$$\mathcal{L}_1 = \frac{1}{4}\, F_\pi^2\, \mathrm{tr}(\nabla_\mu U^\dagger \nabla^\mu U) + \dots , \qquad (2)$$

$$\begin{aligned}
\mathcal{L}_2 &= L_1\langle \nabla_\mu U^\dagger \nabla^\mu U\rangle^2 + L_2\langle \nabla_\mu U^\dagger \nabla_\nu U\rangle\langle \nabla^\mu U^\dagger \nabla^\nu U\rangle \\
&\quad + L_3\langle \nabla_\mu U^\dagger \nabla^\mu U \nabla_\nu U^\dagger \nabla^\nu U\rangle \\
&\quad - iL_9\langle F_{\mu\nu}^{\mathrm{R}} \nabla^\mu U \nabla^\nu U^\dagger + F_{\mu\nu}^{\mathrm{L}} \nabla^\mu U^\dagger \nabla^\nu U\rangle + \dots , \quad (3)
\end{aligned}$$

where I have only displayed the terms which are relevant for later use. In U the pseudoscalar meson fields are encoded. $F_{\mu\nu}^{\mathrm{R,L}}$ denotes the field strength which corresponds to (chirally covariant combinations of) external vector fields v_μ and axial-vector fields a_μ. F_π denotes the pion decay constant (in the chiral limit). I refer to [4] for further details. The four-point meson interaction induced by (3) is depicted schematically in fig. 1b.

As it stands the effective theory (1)-(3) is valid at low energies only. Especially unitarity is not fulfilled. In [5] the inverse amplitude method (IAM) is used to unitarize the effective theory and extend its applicability to the mesonic-resonance region. The IAM is very well suited to recover a resonance from its trace left at low energies [6] and therefore fits perfectly to the philosophy discussed above. In [5] it is demonstrated that the IAM is able to reproduce the scattering data of pions, kaons and etas up to 1.2 GeV including several mesonic resonances. Here I am interested in the large-N_c and chiral limit of the results presented in [5]. It is easy to show that in this limit the mass of the ρ-meson becomes [7]

$$M_V^2 = -\frac{F_\pi^2}{4L_3} . \qquad (4)$$

3 Chiral constituent quark model

As already pointed out it should be reasonable to calculate the coefficients of the effective theory (1)-(3) from a chiral constituent quark model, as the low-energy region is (much) below the quark-antiquark production threshold. In the following I use the quark–Goldstone-boson Lagrangian (in Euclidean space)

$$\mathcal{L}_{\mathrm{quark}} = \bar{q}\,(\gamma_\mu \partial_\mu - MU^{\gamma_5} + \gamma_\mu v_\mu + \gamma_\mu \gamma_5 a_\mu + \dots)\,q . \quad (5)$$

This Lagrangian can be motivated in several ways (*e.g.* [8, 9]). I would like to stress that it is also the simplest model which one can write down which couples quarks to the Goldstone bosons of χSB. The latter are encoded in

$$U^{\gamma_5} = \frac{1-\gamma_5}{2}\,U + \frac{1+\gamma_5}{2}\,U^\dagger . \qquad (6)$$

M denotes the mass of the constituent quark. The dots in (5) denote further couplings to external sources besides

the displayed ones for vector and axial-vector fields. By integrating out the quarks, expanding the obtained effective action in terms of meson field derivatives and finally transforming the result to Minkowski space one arrives at the χPT Lagrangian (1) with predictions for the low-energy constants. This procedure is shown in fig. 1 as the transition from fig. 1c to fig. 1b. For the constants of interest one obtains [9]

$$L_1^{\mathrm{quark}} = \frac{N_c}{384\pi^2} , \qquad L_2^{\mathrm{quark}} = 2L_1^{\mathrm{quark}} ,$$

$$L_3^{\mathrm{quark}} = -4L_1^{\mathrm{quark}} , \qquad L_9^{\mathrm{quark}} = 8L_1^{\mathrm{quark}} . \qquad (7)$$

Note that the results are pure numbers, *i.e.* do not depend on any model-dependent quantities like, *e.g.*, the UV cutoff which regulates UV singular integrals coming from the non-renormalizable Lagrangian (5). The reason is that the loop depicted in fig. 1c is actually UV finite.

4 Resonance saturation model

Assuming that (only) resonances mediate the low-energy interactions of (3) one can write down a Lagrangian which couples resonances to Goldstone bosons [10]. Here I concentrate on the ρ-meson and its interaction Lagrangian

$$\mathcal{L}_{\mathrm{int}} = \frac{F_V}{2\sqrt{2}}\, \mathrm{tr}(V_{\mu\nu} f_+^{\mu\nu}) + \frac{iG_V}{\sqrt{2}}\, \mathrm{tr}(V_{\mu\nu} u^\mu u^\nu) , \qquad (8)$$

where basically u_μ is obtained from U, *i.e.* contains the pseudoscalar fields, while $f_+^{\mu\nu}$ contains the external vector fields (see [10] for details). $V_{\mu\nu}$ denotes the vector meson resonance in the tensor representation. Note that one does *not* assume here that the vector meson couples with the same strength to the external vector field (photon) as it couples to the pseudoscalars (*e.g.* pions). To phrase it differently, universality of the ρ-meson coupling is *not an input* of the resonance saturation model. As I will show below, however, one gains the universality as an *output* of my approach. Integrating out the resonance fields one obtains (3) with predictions for the low-energy coefficients (schematically shown in fig. 1 by the transition from a to b). For the ones governed solely by vector meson exchange one gets [10]

$$L_2^{\mathrm{res}} = \frac{G_V^2}{4M_V^2} , \qquad L_9^{\mathrm{res}} = \frac{F_V G_V}{2M_V^2} . \qquad (9)$$

Note that all the other low-energy constants are additionally influenced by the exchange of mesons with different quantum numbers.

5 Results from matching

In the last sections I have basically collected results from the literature. The new aspect is now that the results from the approaches with hadronic degrees of freedom (sects. 2, 4) are matched to the quark model calculations of sect. 3.

As already pointed out in the introduction, the idea behind that matching is that, on the one hand, the chiral quark model is supposed to give reliable results in the low-energy regime. On the other hand, confinement enforces the formation of resonances (instead of the production of free quarks and antiquarks). These resonances, however, are also visible at low energies, *i.e.* determine the low-energy structure of the strong interaction. From the matching procedure one obtains information about the resonances in terms of quark degrees of freedom.

Even without the results of sect. 4 one obtains from (4) and (7) for the ρ-meson mass:

$$M_V^2 = \frac{24\pi^2 F_\pi^2}{N_c} \,. \tag{10}$$

Using the physical value for the pion decay constant $F_\pi \approx 93\,\mathrm{MeV}$ the ρ-meson mass turns out to be $M_V \approx 826\,\mathrm{MeV}$, already close to the physical ρ-meson mass of $770\,\mathrm{MeV}$. Note that this result was obtained in the chiral and large-N_c limit. Hence pion loops are absent in this framework, *i.e.* I have determined the mass of a bare ρ-meson without its pion cloud. Typically the ρ-pion interaction reduces the bare ρ mass by approximately 5–10% [11,12].

Using in addition (9) yields the coupling constants

$$F_V^2 = 2F_\pi^2 \,, \qquad G_V^2 = \frac{F_\pi^2}{2} \,. \tag{11}$$

In particular, the relation $F_V = 2G_V$ states the universality of the vector meson coupling as can be most easily seen by inspecting (8). This automatically implies that the KSFR relation is fulfilled [13].

The connection of G_V to the usual $\rho\pi\pi$ coupling is provided by

$$g = \frac{G_V M_V}{F_\pi^2} \tag{12}$$

with g defined via the Lagrangian [12]

$$\mathcal{L}_{\mathrm{int}} = \frac{ig}{4}\operatorname{tr}(V^\mu\,[\partial_\mu\Phi,\Phi]) - \frac{g^2}{16}\operatorname{tr}([V^\mu,\Phi]^2)\,, \tag{13}$$

where V^μ is the vector meson resonance in the vector representation and Φ is connected to U via $U = \exp(i\Phi/F_\pi)$. Relation (12) can be obtained by calculating the decay width $\Gamma(\rho \to \pi\pi)$ in both approaches (8) and (13). From (10), (11) one gets

$$g = \sqrt{\frac{3}{N_c}}\,2\pi \tag{14}$$

as compared to the experimental value of 6.05 extracted from the decay width $\Gamma(\rho \to \pi\pi)$ [12].

Finally, properties of the axial-vector meson a_1 can be deduced from the Weinberg sum rules [13,10]:

$$F_V^2 = F_A^2 + F_\pi^2 \,, \qquad M_V^2 F_V^2 = M_A^2 F_A^2 \,. \tag{15}$$

In combination with the previous results this yields the a_1 mass

$$M_A = \sqrt{\frac{3}{N_c}}\,4\pi F_\pi \approx 1169\,\mathrm{MeV} \tag{16}$$

and the coupling of the a_1 to an external axial-vector current

$$F_A = F_\pi \approx 93\,\mathrm{MeV} \tag{17}$$

to be compared to the experimental values $M_{a_1} = 1230 \pm 40\,\mathrm{MeV}$ and $F_{a_1} = 124 \pm 27\,\mathrm{MeV}$ [10].

6 Summary and outlook

I have presented a somewhat indirect way to determine the properties of vector and axial-vector mesons in terms of quark degrees of freedom. The success of the presented approach suggests that it is indeed the phenomenon of χSB which quantitatively determines the properties of the studied mesonic resonances. Confinement enters the framework only qualitatively by demanding that color-white resonances are formed instead of quark-antiquark pairs.

There are things which still have to be clarified: First, I have utilized two different versions of resonance saturation. In the first one (sect. 2) resonances are created from a combination of the two Lagrangians (2) and (3) (cf. [5] for details), while in the second version (sect. 4) the resonances only influence the fourth-order Lagrangian (3). The connection of these two versions has to be studied in more detail. Second, there is a low-energy constant, namely L_{10}, which can be calculated both from the quark model and from the resonance saturation model using vector and axial-vector mesons. It turns out that one needs more than one meson per channel to achieve an agreement between the two calculations [14].

I expect that the presented framework can be extended from the vacuum case studied here to the case of a medium with finite temperature and/or quark density. This should provide interesting insight in the in-medium changes *e.g.* of the ρ-meson mass, its coupling to pions and photons and possible differences between longitudinal and transverse ρ-mesons.

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